

Shannon Entropy as a Measure of Risk Based on Modern Portfolio Theory

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ABSTRACT

The proxy for risk is variance based on modern portfolio theory. The stock returns typically follow asymmetric distribution leading to implausible estimation of risk. This study replaces variance with Shannon Entropy as a measure of risk and applies it to modern portfolio theory. It accommodates asymmetric distributions, and it is fully nonparametric. The study employs portfolio optimization based on variance and Shannon Entropy for MINT, Visegrád group, and CIVETS markets. The results are compared using tracking error. The study reports that the mean-variance portfolio optimization tracking errors are higher than those of the Shannon Entropy portfolio optimization. This implies that the portfolio estimated, using the latter, closely follows its benchmark. The study benefits investors in better understanding the risk assessment and making plausible decisions based on return-entropy portfolio optimization adjusted for the desirable benchmark index.

Keywords: *Shannon Entropy, variance, modern portfolio theory, tracking error*

INTRODUCTION

Risk mitigation in the investment arena is an essential diversification strategy based on modern portfolio management. The proxy for risk measurement is variance/covariance. Therefore, the portfolio minimization method requires does not accommodate asymmetric distributions. In the real world, stock returns are typically asymmetric/non-normally distributed, thus rendering the variance a questionable measure of risk. Especially in extreme positions, this may lead to errors that substantially change the optimal portfolio weights (Díaz et al., 2024).

Keeping together the modern portfolio theory framework, entropy may be an alternative to variance/covariance as an alternative measure of risk. The concept of entropy statistically assumes the probabilistic measure of uncertainty or disorder. The amount of information is considered the random variable of entropy. The more the order will be if the information it contains is more thus,

rendering entropy a useful tool that quantifies the amount of information in a random variable and also provides a good measure of the disorder present in a probability (Mercurio et al., 2020).

Replacing variance/covariance by entropy is a measure of risk that does not require the normality assumption, can accommodate asymmetric distributions, and is fully nonparametric. All main difficulties with variance-based portfolio optimization may be solved as entropy-based methods provide greater diversification. Secondly, the dependence structure is maintained because the entropy is not based on the mean. Thirdly, the optimal solution is more stable under the adjustments to investors' views. Lastly, the joint entropy dependence structure can be automatically captured in the objective function, leading to no pre-calculation of covariance between stock returns (Mercurio et al., 2020).

Roll (1992) and Jorion (2002) are the pioneers who use tracking errors for portfolio optimization solutions. They analytically trace the trade-off between active risk-taking and expected excess returns. Chan and Lakonishok (1993), and Chan, Karceski, and Lakonishok (1999) advocate the use of tracking error either as an investment goal or it can be an investment constraint. Clarke et al. (2006) investigate different active strategies with or without weight and/or risk constraints. Tracking errors, therefore, can be effective in assessing the performance of a portfolio.

The study replaces the risk assessment based on variance by Shannon Entropy. Furthermore, the study focuses on economic blocks instead on individual stocks. The focus of economic blocks provides validation for other economies to follow both on individual stocks, mutual funds and help in international diversification. The entropy-based portfolio optimization is applied to three economic blocks, namely MINT countries (Mexico, Indonesia, Nigeria, and Turkey), Visegrád group (V4) (Czech Republic, Poland, Hungary, and Slovakia) and CIVETS countries (Colombia, Indonesia, Vietnam, Egypt, Turkey and South Africa). Furthermore, the MSCI Emerging Market index tracks the portfolio's performance based on tracking errors. This makes this study one of its kind. Section one of this study is an introduction, section two is the literature review, section three is data and methodology, section four is results and discussion, and the last section is the conclusion.

LITERATURE REVIEW

Mean-Variance Portfolio Optimization and its Criticism

Although Markowitz's (1952) mean-variance portfolio optimization has been used extensively in the financial literature, as Mercurio, Wu, and Xie (2020) explained, Markowitz's mean-variance methodology assigned large weights to risky assets, this usually occurs when the investor changes the risk parameter for higher returns. This leads to sparse solutions that result in negligible diversification. Due to this reason, Markowitz's method fails to achieve the goal of diversification (Mercurio et al., 2020).

Furthermore, one of the significant consequences of Markowitz's model is that it relies on the normality assumption. In their study, Xiong et al. (2011) suggested that most portfolios and

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asset classes are non-normally distributed. They are not symmetric. Due to these properties of returns, variance is considered a poor measure of risk. Furthermore, the covariance matrix is calculated using the historical returns, but they do not always indicate future returns. Therefore, investors forecast expected returns, and their views of expected returns are used instead. So, estimating the covariance matrix is challenging (Xiong et al., 2011; Mercurio et al., 2020).

Background of Shannon Entropy as a Measure of Risk

In 1865, Rudolf Clausius developed the concept of entropy. He described it as, in the thermodynamic system, the entropy is a measure of the amount of energy, cited by (Fang et al., 1997). Through the advancement of statistical mechanics, the concept of entropy was extended. 1948, Claude Shannon introduced entropy into information theory, as Law & Zaniolo (2005) cited. It was argued by Shannon (1948) that the concept of entropy can be extended to those fields of science in which the probabilities are calculated. Ever since, entropy has become the key principle of information theory. Modern financial economics and econophysics use many concepts from information theory.

Philippatos and Wilson (1972) first used entropy in the portfolio optimization problem. While comparing the behaviour of traditional standard deviation with entropy, they concluded that entropy is superior to standard deviation in explaining risk. While comparing variance with entropy, Ebrahimi et al. (1999) suggested that variance measures the concentration only around its mean value. On the other side, entropy measures density diffuseness regardless of the location of the concentration. In the context of statistics, entropy is not a frequentist mean-centred measure; entropy considers the whole empiric distribution without relying upon some specific statistical moment.

Backus, Chernov and Zin (2014) explained that entropy is regarded as a measure of dispersion in financial literature. Entropy is defined as Information entropy applied to a probability vector and reflecting the amount of uncertainty or randomness underlying that specific probability distribution. Shannon entropy represents the measure of the Number of "choices" involved while selecting an event. Alternatively, the amount of certainty about an outcome can be said. In probability theory, entropy is considered as the measure of uncertainty. Moreover, in risk measuring, entropy has also been an alternative to variance (Campbell, 2015). As stated earlier, entropy measures a random variable's dispersion. Moreover, it is also used to measure uncertainty.

Entropy as a Measure of Risk in Portfolio Diversification

Many scholars have applied entropy as a measure of diversity (Philippatos & Wilson, 1972; Hoskisson et al., 1993; Samanta & Roy, 2005; Dionisio et al., 2006; Bera & Park, 2008; Jana et al., 2009; Usta & Kantar, 2011). They stated that there exists a positive relationship between entropy and portfolio diversification. In multi-objective portfolio optimization models, Samanta and Roy (2005), Bera and Park (2008), Jana et al. (2009) and Usta and Kantar (2011) used entropy as an objective function.

Bera and Park (2008) presented an asset allocation model using entropy and cross-entropy measures. They explained that the speciality of their proposed model is that it generates well-diversified portfolios. Furthermore, using entropy as an objective function to allocate optimal weights to the assets in the portfolio states that the assigned weights are non-negative. Therefore an entropy-based model accounts for no short selling, which the risk-averse investors usually prefer (Jana et al., 2009).

Furthermore, Usta and Kantar (2011) added Shannon entropy as a measure of diversity to the original mean-variance-skewness model and presented the mean-variance-skewness entropy model (MVSEM) for portfolio selection. In contrast, Zhang et al. (2012) in their study tried to address the limitations in the models presented by Jana et al. (2009) and Kapur (1989). They presented a new model, namely, the mean-semivariance-entropy model. Their model has four criteria: risk, return, degree of portfolio diversification, and transaction cost.

Moreover, the derivation of the portfolio modelling was considered under a fuzzy situation by some scholars such as Smimou et al. (2007). Conversely, Huang (2008) presented a fuzzy mean-entropy model to construct a mean-entropy frontier. Rödder et al. (2010), under a maximum and minimum relative entropy, presented a new model to determine the portfolio weights using a rule-based inference mechanism. Additionally, some other researchers, such as Zhang et al. (2012), also proposed a model for multi-period portfolio selection problems with the inclusion of transaction cost. Zhou et al. (2013) presented a model that includes information entropy-incremental entropy-skewness, whereas the portfolio risk was measured by information entropy.

Since the calculation of entropy is itself complex, several studies are dedicated to improving entropy calculation. Similarly, Mercurio and Wu (2020) recently presented a return entropy portfolio optimization method. Mercurio, Wu, and Xie (2020) in their study used the minimum entropy method and presented an approach for portfolio optimization, which is known as return-entropy portfolio optimization (REPO). They explained that the speciality of return entropy portfolio optimization methodology is that, compared to the other methods, it measures the resultant portfolio entropy directly within the objective function, which uses combinatorial generating functions. It makes the methodology much easier as there is no need to calculate joint entropy.

As stated above, Markowitz's (1952) mean-variance portfolio optimization approach has certain difficulties. However, Mercurio et al. (2020) presented a portfolio optimization approach known as return-entropy portfolio optimization (REPO). They stated that while optimizing a portfolio, all those shortcomings can be solved if the variance is replaced with entropy. As Mercurio et al. (2020) explained, the return-entropy portfolio optimization approach has the advantage of selecting low-risk portfolios that provide a greater level of diversification than those selected by the mean-variance portfolio optimization method. It can also preserve the dependence structure. As Mercurio et al. (2020) REPO approach directly calculates the portfolio entropy into its objective function according to the weights of each security, that maintains the dependence structure equilibrium as per the assumptions of investor or proper to the history. So, the

dependence structure will not be affected whenever input changes are made. Unlike variance, the value of entropy does not depend upon the mean value.

Furthermore, whenever investor views are incorporated, optimal solutions found with REPO are far more stable than the ones found with mean-variance portfolio optimization. So, it is optional to estimate the expected return accurately. Also, entropy does not have any distribution assumption. It does not require the data to be normally distributed. It can be applied to asymmetric or non-normally distributed data. So, the return-entropy portfolio optimization approach is an entirely nonparametric statistical method. Moreover, the risk is directly calculated in the objective function, so there is no need to calculate the covariance matrix or variance of the securities beforehand (Mercurio et al., 2020).

Data Description

The study uses monthly closing prices of international equity indexes of three economic blocks, namely MINT countries (Mexico, Indonesia, Nigeria, and Turkey), Visegrád group (V4) (Czech Republic, Poland, Hungary, and Slovakia), and CIVETS countries (Colombia, Indonesia, Vietnam, Egypt, Turkey, and South Africa). These are the rising blocks with most developing and emerging countries in it. Moreover, the countries are dispersed and located in almost all the continents, yielding the maximum benefit of international diversification. Moreover, the study is conducted on three economic blocks, namely MINT countries (Mexico, Indonesia, Nigeria, and Turkey), Visegrád group (Czech Republic, Poland, Hungary, and Slovakia), and CIVETS countries (Colombia, Indonesia, Vietnam, Egypt, Turkey, and South Africa).

O'Neill (2013) recognized an emerging power block as "MINT" countries: Mexico, Indonesia, Nigeria, and Turkey. He stated, "Mexico, Indonesia, Nigeria, and Turkey all have favourable demographics for at least the next 20 years, and their economic prospects are interesting."

Goldman Sachs 2012 stated that Mexico was ranked as having the 14th largest economy in the world, Indonesia ranked as the 16th largest world economy, Turkey's economy ranked 17th, and Nigerian economy ranked 39th. According to the World Bank (2014) data on the GDP in US dollars, the rankings of the Nigerian economy have improved; it was ranked 21st, while Turkey was ranked as the 17th best economy in the world. Furthermore, the Indonesian economy ranked 16th, and the Mexican economy ranked as the 15th largest economy in the world. The 2050 forecast predicts Mexico as the 8th largest economy globally, followed by Indonesia as the 9th largest, Nigeria as the 13th largest, and Turkey as the 14th largest economy by the end of 2050.

V4 consists of four Central European countries: the Czech Republic, Slovakia, Hungary, and Poland. The Visegrád countries are the most advanced among the recent European Union members. According to the World Bank's 2014 data, these countries are now considered "high-income countries" with a gross national income (GNI) per capita above \$12,735. Moreover, they are ranked the highest in the human development index.

According to 2015 available data, the Czech Republic is the most industrialized economy, with a gross domestic product per capita PPP of \$32,622. Slovakia and Poland follow, and Hungary has a gross domestic product per capita of \$26,941. If these four nations were counted as a single state, they would be considered the fifth-largest economy in Europe and the 12th-largest economy globally.

CIVETS countries comprise six emerging economies: Colombia, Indonesia, Vietnam, Egypt, Turkey, and South Africa. As per the data from 2009, the total population of CIVETS is 577.3 million. Indonesia is the most populous, with 240.3 million population, followed by Vietnam, 88.6 million; Egypt, 78.9 million people; Turkey, 76.8 million; South Africa, 49.1; and Colombia, 43.7 million. The average per capita income in CIVETS is 6163 in international dollars ((I\$); in Turkey, the average per capita income is 9833 I\$; in South Africa, it is 7816 I\$, Colombia 7714 I\$, 4722 I\$ in Egypt, 3957 I\$ in Indonesia and 2934 I\$ in Vietnam as cited by (Korkmaz et al., 2012).

Although these three economic blocks are essential for the global economy and the whole world, there is a massive opportunity for portfolio diversification from these blocks of emerging markets. These economies are projected to reflect steady economic growth and domestic demand in the next decade. Therefore, investigating the diversification of portfolios among these is an important step toward potential growth opportunities.

The collected stock indices data for MINT countries are from the BMV IPC index for Mexico, IDX composite index for Indonesia, NSE All Share for Nigeria, and BIST100 index for Turkey. The data for the Visegrád group is collected from the Prague stock exchange for the Czech Republic, WIG for Poland, BUX for Hungary, and SAX for Slovakia. The data for CIVETS countries are collected from COLCAP for Colombia, IDX composite index for Indonesia, VN index for Vietnam, EGX 30 for Egypt, BIST100 Index for Turkey, and FTSE/JSE Top40 Index for South Africa. The MSCI Emerging Market index was used to track the performance of the portfolio.

RESEARCH METHODOLOGY

The risk associated with each country in the portfolio was calculated with both the risk measures, i.e. variance and Shannon entropy, to compare the relative risk rankings obtained with each measure. As Markowitz (1952) defined risk with the variance σ^2 , the variance was used to measure the risk. The formula to estimate variance is presented below:

$$\sigma^2 = \frac{\sum(x_i\bar{x})^2}{n-1} \dots\dots\dots(1)$$

Where x_i represents the return of country I, \bar{x} represents the mean return of country and n . Represents the total Number of observations.

Mercurio and Wu (2020) defined risk with Shannon entropy, so entropy was also used to measure the risk. The formula to estimate Shannon entropy is presented below,

$$H(X) = E(-\log P(X)) = - \sum_{i=1}^n P(x_i) \log P(x_i) \dots \dots \dots (2)$$

where $P(x_i)$ represents the probability of event x . Natural logarithm was used; therefore, entropy was in natural units.

If a portfolio consists of n assets. Their expected future returns will be expressed as $E(R_1), \dots, E(R_n)$. So, the goal of the portfolio selection problem is to construct an optimal portfolio R_p , which allocates weights $w_1, \dots, \text{and } w_n$ that represent the percentage of capital to be invested in each asset. Consequently, the objective function will minimize the portfolio's risk and maximize the expected returns. In the methodology presented by Markowitz (1952), risk is measured by the variance. The multi-objective function used by Markowitz's mean-variance portfolio optimization (MVPO), which minimizes variance and maximizes expected future return, is defined as follows.

$$\text{minimize } \text{Var}(R_p) = w_1^2 \sigma_1^2 + \dots + w_n^2 \sigma_n^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ji} \dots \dots \dots (3)$$

$$\text{maximize } E(R_p) = w_1 E(R_1) + \dots + w_n E(R_n)$$

$$\text{subject to } w_1 + \dots + w_n = 1, \quad w_i \geq 0 \forall i,$$

whereas σ_k^2 denotes the variance of R_k , and $R_p = w_1 R_1 + \dots + w_n R_n$

Mercurio, Wu, and Xie (2020) explained in their study that if X is a discrete random variable and it has a probability mass function $P(\cdot)$ which can take possible values x_1, \dots, x_n . So, the Shannon entropy represents the average amount of information produced by X . Shannon entropy is denoted by H . It is defined as follows,

$$H(X) = E(-\log P(X)) = - \sum_{i=1}^n P(x_i) \log P(x_i) \dots \dots \dots (4)$$

According to Mercurio, Wu, & Xie (2020), R denotes the return of an asset and $r = (r_1, \dots, r_T)$ denotes its historical observation (actual returns). These historical observations are divided into m distinct probability states, A_1, \dots, A_m which has endpoints $[a_0, a_1], \dots, [a_{m-1}, a_m]$ respectively. Such that each r_j belongs to only one portion i.e. $a_{k-1} < r_j \leq a_k$, for some integer k in $1, \dots, m$. So, $\{R \in A_k\}$ is the probability of each event which is calculated over T period by the following equation.

$$\hat{f}_T(r; k) = \frac{1}{T} \sum_{j=1}^T I(a_{k-1} < r_j \leq a_k) \approx P(R \in A_k) \dots \dots \dots (5)$$

A portfolio has only two assets, so R_1 and R_2 denote their returns. Then $r_1 = (r_{11}, \dots, r_{1T})$ and $r_2 = (r_{21}, \dots, r_{2T})$ represents their historical observations over T time units. Portfolio weights are represented by w_1, w_2 , such that $w_1 + w_2 = 1$. So the probability generating function g is defined as follows; $g(x; w_1r_1 + w_2r_2) = \frac{1}{T} \sum_{j=1}^T x^{\{k: a_{k-1} < w_1r_{1j} + w_2r_{2j} \leq a_k\}} \dots \dots \dots (6)$

For k such that $\{a_{k-1} < w_1r_{1j} + w_2r_{2j} \leq a_k\}$

Whereas x^k calculates the empirical probability of event $\{R_p \in A_k\}$, $\hat{f}_T(r_1, r_2; k)$. The coefficient of x^k for each polynomial term can be extracted by dividing the k_{th} derivative of g at $x = 0$ i.e. $g^{(k)}(0)$ and dividing it by $k!$. It is defined as;

$$\frac{g^{(k)}(0)}{k!} = \hat{f}_T(r_1, r_2; k) = \frac{1}{T} \sum_{j=1}^T I(a_{k-1} < w_1r_{1j} + w_2r_{2j} \leq a_k) \approx P(R_p \in A_k) \dots \dots \dots (7)$$

Subsequently, these probabilities are substituted in the formula of Shannon entropy (Shannon, 1948) for m total probability states. HRP represents portfolio entropy(), and substituted probabilities in the formula of Shannon entropy are defined as;

$$H(R_p) = - \sum_{k=1}^m \hat{f}_T(r_1, r_2; k) \log \hat{f}_T(r_1, r_2; k) \dots \dots \dots (8)$$

As mentioned above can be extended to n asset case, R_1, \dots, R_n and their actual returns over T time units are $r_i = (r_{i1}, \dots, r_{iT})$, $i = 1, \dots, n$ and the portfolio weights are w_1, \dots, w_n such that $w_1 + \dots + w_n = 1$. Not too small interval sizes (the choice of m) nor huge interval sizes were selected, as it effects the results of entropy calculation. Particular portfolio entropy is calculated by the following portfolio entropy objective function for m unique probability states, A_k

$$H(R_p) = - \frac{g'(0)}{1!} \log \left(\frac{g'(0)}{1!} \right) - \frac{g''(0)}{2!} \log \left(\frac{g''(0)}{2!} \right) - \dots - \frac{g^m(0)}{m!} \log \left(\frac{g^m(0)}{m!} \right) \dots \dots \dots (9)$$

Each term in the above objective function represents its respective term in the entropy function. This objective function is subsequently minimized in the optimization problem. So, return-entropy portfolio optimization (REPO) Mercurio et al. (2020) uses a multi-objective function. It minimizes entropy and maximizes expected returns, defined as follows.

$$\text{minimize } H(R_p) = - \sum_{k=1}^m \frac{g^{(k)}(0)}{k!} \log \left(\frac{g^{(k)}(0)}{k!} \right) \dots \dots \dots (10)$$

$$\text{maximize } E(R_p) = w_1 E(R_1) + \dots + w_n E(R_n)$$

$$\text{subject to } w_1 + \dots + w_n = 1, \quad w_i \geq 0 \forall i,$$

For $R_p = w_1R_1 + \dots + w_nR_n$ and the k th derivative at $x = 0$ of the probability generating function:

$$g(x; w_1r_1 + \dots + w_nr_n) = \frac{1}{T} \sum_{j=1}^T x^{\{k: a_{k-1} < \sum_{i=1}^n w_i r_{ij} \leq a_k\}}$$

Returns are computed as;

$$r_{ij} = \frac{P_{ij}}{P_{i,j-1}} - 1$$

r_{ij} represents the per cent return on security i in period j . Whereas P_{ij} denotes the price of security i in period j . The methodology presented by Mercurio et al. (2020) was followed in this study. Percent returns were calculated by using interval sizes of one basis point. And minimum and maximum across all the returns was used as support boundaries, i.e.

$$[a_{min}, a_{min} + 1], \dots, [-2, -1], [-1, 0], [0, 1], [1, 2], \dots, [a_{max} - 1, a_{max}]$$

All are in units of basis points.

Bera and Park (2008) suggested using Shannon (1948) to measure portfolio diversification. It is defined as follows,

$$SE(\pi) = - \sum_{j=1}^n \pi_j \log(\pi_j) \dots \dots \dots (11)$$

Where as π_j represents the weights of assets.

Tracking Error

Tracking error represents the difference between the portfolio's return and its benchmark (index). It is often used to evaluate the performance of portfolio managers. The portfolio manager has performed well if the portfolio can offer the same return earned on the benchmark. So, the lower the tracking error, the better the performance. Furthermore, low tracking error implies that the portfolio closely follows its benchmark. However, on the other side, high tracking error indicates poor performance of a portfolio manager. Therefore, tracking error gives an idea to the investor about how "tight" the portfolio is around its benchmark or the volatility of the portfolio in comparison to the benchmark.

Absolute deviation tracking error and standard deviation tracking error were used in the study. According to the methodology proposed by Chu (2011), the following is the definition

presented by Pope and Yadav (1994). This is the absolute difference in the returns of the portfolio and the returns of the benchmark. It represents the extent to which the portfolio returns deviate from the benchmark's returns over a given period. It is calculated as follows;

$$T.E_{AD,j} = \frac{\sum_{j=1}^n |e_{j,t}|}{n} \dots \dots \dots (12)$$

where $T.E_{AD,j}$ = Tracking error absolute deviation, $e_{i,t} = R_{p,t} - R_{b,t}$ (Difference between Portfolio return and benchmark index return) and n = Number of observations

Another way to find tracking errors is to estimate the standard deviation of the difference between the portfolio's returns and the benchmark's returns. It is calculated as follows,

$$T.E_{SD,j} = \sqrt{\frac{\sum_{t=1}^n (e_{Ej,t} - \bar{e}_j)^2}{n - 1}} \dots \dots \dots (13)$$

where, $T.E_{SD,j}$ = Standard deviation of T.E, $e_{i,t} = R_{p,t} - R_{b,t}$ (Difference between Portfolio return and benchmark index returns), \bar{e}_j = average of $e_{i,t}$ and n = Number of observations.

RESULTS AND ANALYSIS

Risk is calculated using variance and Shannon entropy for each country in three economic blocks. Risk estimations, along with the average return, are presented in Tables 1 to 3.

Table 1: Mean, Variance and Shannon entropy of MINT Countries

MINT Countries	Mean	Variance	Shannon Entropy
Mexico	0.204	14.315	2.573
Indonesia	0.521	17.505	2.593
Nigeria	0.198	36.697	3.067
Turkey	0.741	42.305	3.147

Table 1 presents the mean, variance, and Shannon entropy of MINT Countries. The average return of all MINT countries is estimated to be positive, with values of 0.204, 0.521, 0.198, and 0.741 for Mexico, Indonesia, Nigeria, and Turkey, respectively. Turkey ranked as the highest return country among MINT countries, with an average return of 0.741. In contrast, Nigeria has the lowest return, with a value of 0.198.

The risk estimations measured with both the risk measures (variance and Shannon entropy) are also presented in Table 1. Mexico was the best country with the lowest risk at Bahria University Journal of Management and Technology (BJMT).2025, Volume 8, Issue 1

14.315² and 2.573, estimated with variance and entropy, respectively. In contrast to that, Turkey is the riskiest country among MINT countries. Turkey has the highest risk of 42.305 (variance) and 3.147 (entropy). The risk estimation for Indonesia is 17.505 and 2.593. Moreover, for Nigeria is 36.697 and 3.067.

Furthermore, the ranking in terms of variance and entropy is the same for MINT countries. The trend here also validates the assumption that the higher the return, the higher the entropy. The correlation coefficient between the return and the entropy is 0.335983.

Table 2: Mean, Variance, and Shannon entropy of Visegrád Group (V4)

V4 Countries	Mean	Variance	Shannon Entropy
Czech Republic	-0.113	20.010	2.720
Slovakia	0.389	14.157	2.521
Hungary	0.505	28.514	2.918
Poland	0.289	20.826	2.811

Table 2 illustrates each country's return, variance, and Shannon entropy in the Visegrád Group (V4). The average return for Slovakia, Hungary, and Poland is positive (0.389, 0.505, and 0.289, respectively) but negative in the Czech Republic (-0.113). So, Hungary is the most profitable country, while the Czech Republic has the lowest return in the sample.

Similarly, risk for each country is estimated using both risk measures: variance and Shannon entropy. Slovakia has the lowest variance and entropy, at 14.157² and 2.521, respectively. At the same time, Hungary is estimated as the riskiest country across the Visegrád group, at 28.514² and 2.918 with variance and entropy, respectively. The estimated variance and Shannon entropy for the Czech Republic are 20.00² and 2.720, and for Poland, they are 20.826² and 2.811, respectively.

The risk estimations shown in the table show that the relative risk ranking in terms of variance and entropy is the same in the case of the Visegrád group as well. Moreover, the trend here also validates the assumption that the higher the return, the higher the entropy. The correlation coefficient between return and entropy in the sample of the Visegrád group is 0.181819.

Table 3: Mean, Variance and Shannon entropy of CIVETS Countries

CIVETS Countries	Mean	Variance	Shannon Entropy
Colombia	-0.203	22.580	2.759
Indonesia	0.521	17.505	2.636

Vietnam	0.651	35.138	2.989
Egypt	0.791	75.423	3.255
Turkey	0.741	42.305	3.162
South Africa	0.716	16.102	2.693

Table 3 presents each CIVETS country's average return, variance, and Shannon entropy. The average return in Indonesia, Vietnam, Egypt, Turkey, and South Asia is positive (0.521, 0.651, 0.791, 0.741, and 0.716, respectively) but negative in Colombia (-0.203). So, Egypt ranked as the highest return country across CIVETS. In addition, the lowest average return is estimated in Colombia.

Table 3 also presents the risk estimations of each CIVETS country measured using variance and entropy. Variance ranked South Africa as the best country with the lowest risk, 16.102, whereas entropy ranked Indonesia as the safest, at 2.636. According to both risk measures, Egypt is ranked as the riskiest country among CIVETS, at 75.423 and 3.255, estimated with variance and entropy, respectively.

The estimated variance for Colombia is 22.580, for Indonesia is 17.505, for Vietnam is 35.138, for Turkey is 42.305, and for South Africa is 16.102. In contrast to variance, the estimated Shannon entropy for Colombia is 2.759, for Vietnam is 2.989, for Turkey is 3.162 a, and for South Africa is 2.693.

It can be seen from the risk estimations shown in the table that the relative risk ranking in terms of variance and entropy are almost identical, except for Indonesia and South Africa. Indonesia and South Africa have switched their positions in risk rankings. Variance ranked Indonesia 5th and South Africa as the least risky country. In contrast, entropy ranked South Africa as the 5th and Indonesia as the safest country. Furthermore, it is noted that the trend here also validates the assumption that the higher the return the higher the entropy. The correlation coefficient between return and entropy in the sample of CIVETS countries is 0.466804.

Optimal weights for each country in three economic blocks—MINT, Visegrád group, and CIVETS countries—are calculated according to the algorithms defined in the methodology section. Minimum risk portfolios and optimal solutions are presented in Tables 4 to 6 using both methods: Markowitz's (1952) mean-variance portfolio optimization (MVPO) and Mercurio et al.'s (2020) return entropy portfolio optimization (REPO).

Table 4: Optimal Minimum Risk Portfolio Using MVPO and REPO for MINT countries

Method	Expected Return	Minimum Risk	Weights				Diversification level
			Mexico	Indonesia	Nigeria	Turkey	

MVPO	0.328	10.606	46.8%	34.0%	16.0%	3.3%	1.127
REPO	0.470	2.620	15.0%	25.0%	25.0%	35.0%	1.345

Table 4 shows the minimum risk portfolios achieved by both methods. Notably, the minimum objective of REPO and MVPO is at the different expected returns and optimal solutions. In the case of mean-variance portfolio optimization, the minimum risk portfolio is achieved at 10.606² with the expected return of 0.328. Figure 1 gives a clear picture of the allocation of weights; here, most weights are assigned to Mexico (46.8%), followed by Indonesia (34%) and Nigeria (16%), with a tiny percentage to Turkey (3.3%).

In contrast to MVPO, return entropy portfolio optimization achieved its minimum risk portfolio at 2.620 with an expected return of 0.470. It was also found that REPO offered a greater return of 0.142. Moreover, the optimal solutions found with REPO offer better diversification as it did not assign large weights to some of the countries in the MINT region. Optimal weights allocated with REPO for Mexico, Indonesia, Nigeria, and Turkey are 15%, 25%, 25%, and 35% respectively. A pie chart, figure 2, represents optimal weights found with REPO.

Shannon entropy, explained in the literature and methodology section, suggests that it has more excellent empirical support for portfolio diversification. Therefore, a separate measure for portfolio diversification is used in the current study, along with the pie charts. So, Table 4 also shows the diversification level achieved with both methods.

It is noted that the optimum minimum risk portfolio achieved with MVPO provides a diversification level of 1.127 only. However, on the other side, the optimal solutions found with REPO proved the diversification of 1.345. So, REPO provides a greater level of diversification by 21.8%. Hence, from both the pie charts and the Shannon entropy, it is found that the REPO method gives optimal solutions that are far more diversifiable; therefore, it is the winning method.

Table 5: Optimal Minimum Risk Portfolio Using MVPO and REPO for Visegrád group

Method	Expected Return	Minimum Risk	Weights				Diversification level
			Czech Republic	Slovakia	Hungary	Poland	
MVPO	0.277	9.437	21.07%	55.94%	7.56%	15.42%	1.137

REPO	0.260	2.466	24.62%	22.05%	22.14%	31.20%	1.376
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Table 5 presents minimum risk portfolios achieved by mean-variance portfolio optimization and return entropy portfolio optimization for the Visegrád group. Again, both methods estimated their minimum risk portfolios at different expected returns and optimal solutions. For Markowitz's (1952) method, the minimum risk portfolio is achieved at 9.437² which offers an expected return of 0.277.

The optimal weights found with MVPO are 21.07%, 55.94%, 7.56%, and 15.42% for the Czech Republic, Slovakia, Hungary, and Poland, respectively. However, again, Markowitz's (1952) mean-variance portfolio optimization produced the solutions for which it is usually criticized: concentrated weights on a few assets by assigning them a large percentage. Here, in the case of the Visegrád group, large weights are assigned to Slovakia, followed by the Czech Republic. So, more than 50% of the investor's wealth is invested in a single stock, i.e. Slovakia. The pie chart in Figure 3 interprets the optimal weights allocated using the mean-variance method. On the other hand, the Mercurio et al. (2020) return entropy portfolio optimization method achieved its minimum risk portfolio at 2.466 with an expected return of 0.260. Here, the weights are allocated more evenly, as seen clearly from the pie chart (Figure 4). The Optimal weights assigned by REPO to the Czech Republic, Slovakia, Hungary, and Poland are 24.62%, 22.05%, 22.14%, and 31.20%, respectively.

From the pie charts and the diversification level estimated with Shannon entropy, it is again noteworthy that the optimal solutions found with REPO offer a better diversification level at 1.376 than those found with MVPO, which only offers a diversification level of 1.137. Thus, REPO offers a higher diversification by 23.9%.

Table 6: Optimal Minimum Risk Portfolio Using MVPO and REPO for CIVETS Countries

Method	Expected Return	Minimum Risk	Weights						Diversification level
			Colombia	Indonesia	Vietnam	Egypt	Turkey	South Africa	
MVPO	0.50	11.03	14.70%	37.09%	8.19%	0.00%	0.00%	40.02%	1.221
REPO	0.527	2.504	15.78%	18.65%	20.95%	0.00%	22.54%	22.09%	1.601

Table 6 illustrates the optimal solutions and minimum risk portfolios for CIVETS countries. Results are estimated by mean-variance portfolio optimization (MVPO) (Markowitz, 1952) and return entropy portfolio optimization (REPO) (Mercurio et al., 2020), described in the methodology section. It is shown in table 6 that the minimum objective found by both methods is achieved at the different expected return and optimal solutions. In the case of mean-variance portfolio optimization, the minimum risk portfolio is achieved at 11.03² with the expected return

of 0.50. MVPO assigned a large weight percentage to a few economic markets in CIVETS, such as South Africa (40.02%) and Indonesia (37.09%). The remaining countries are assigned very small percentages of weight in the optimal portfolio, such as 14.70% to Colombia, and 8.19% to Vietnam, with no weight allocation to Egypt and Turkey.

Compared to MVPO, REPO achieved its minimum risk portfolio at the risk level 2.504 with an expected return of 0.527. The Optimal weight allocated with REPO for Colombia is 15.78%, for Indonesia 18.65%, for Vietnam 20.95%, for Turkey 22.54%, and for South Africa 22.09%. No percentage of weight was allocated to Egypt.

Unlike MVPO, REPO does not concentrate on fewer assets. It assigns weights to provide a greater level of diversification. Therefore, optimal solutions found with MVPO provide a diversification level of only 1.221, whereas those found with REPO provide a greater level of diversification at 1.601, which is 38% higher than MVPO.

Consequently, the optimum solutions found with both methods differ greatly. Individual country holdings in the portfolio changed significantly while using variance in the objective objection compared to entropy in the objective function. In addition to assigning different weights to each economic market in the portfolio, both methods differ greatly regarding providing a diversification level. In each case, the return-entropy (REPO) method offered greater diversification in the portfolio.

Furthermore, the results are in line with the previous studies of Philippatos and Wilson (1972), Bera and Park (2008), Jana et al. (2009), Samanta and Roy (2005), and Usta and Kantar (2011), and Mercurio et al. (2020) who used entropy as an objective function. Moreover, explains that it generates well-diversified portfolios.

Tracking error represents the difference between the portfolio's return and its benchmark (index). It is often used to evaluate the performance of portfolio managers. The portfolio manager has performed well if the portfolio can offer the same return earned on the benchmark. So, the lower the tracking error, the better the performance. Furthermore, low tracking error implies that the portfolio closely follows its benchmark. However, on the other side, high tracking error indicates poor performance of a portfolio manager. Therefore, tracking error gives an idea to the investor about how "tight" the portfolio is around its benchmark or the volatility of the portfolio in comparison to the benchmark. Absolute deviation tracking error and standard deviation tracking error are calculated according to the methodology used by Chu (2011). The results of tracking errors for MINT, V4 and CIVETS countries are presented in Table 7.

The tracking error of each portfolio is presented in Table 7. Using Markowitz's (1952) methodology, the tracking error absolute deviation ($T.E_{AD,j}$) for MINT countries is 2.91%; for the Visegrád group, it is 3.39%; and for CIVETS countries, it is 2.52%. On the other side Mercurio & Wu's (2020) methodology reported lower tracking errors. The tracking error absolute deviation ($T.E_{AD,j}$) for MINT countries is 2.79%; for the Visegrád group, the tracking error absolute deviation ($T.E_{AD,j}$) is 2.74%, and for CIVETS countries, it is 2.42%.

Here, Markowitz's (1952) mean-variance portfolio optimization is higher than the tracking errors of Mercurio & Wu's (2020) return entropy portfolio optimization. This implies that the portfolio estimated using REPO closely follows its benchmark and is tighter toward its benchmark. So, from the viewpoint of the investors, the portfolio optimized using the algorithm of return entropy portfolio optimization provides an efficient tracking of their benchmark index. The finding of the study is compatible with other studies such as Giunta et al. (2024) which uses entropic value at risk (EVaR) as a measure of risk on both developed S&P 500 and Euro Stoxx 50) and developing markets (BIST 100 and Bovespa) markets.

The report that EVaR measures effectively identifies the optimal portfolios and la an important role for mitigating negative net performances during low returns or high turnover.

Table 7: Tracking Error of 2MVPO and REPO for MINT, V4 and CIVETS

Portfolio	Minimum Variance Portfolio		Minimum Entropy Portfolio	
	Tracking Error Absolute Deviation	Tracking Error Standard Deviation	Tracking Error Absolute Deviation	Tracking Error Standard Deviation
MINT	2.91%	3.62%	2.79%	3.62%
V4	3.39%	4.35%	2.74%	3.58%
CIVETS	2.52%	3.23%	2.42%	3.28%

Similarly, Bonacic et al. (2024) compares Markowitz framework with fuzzy entropy variants for indexes from the USA, EU, China, and Japan. They report that fuzzy entropy variants provide a better representation of the ambiguity of the information according to the investors' deadline. Though the risk measures used in both the studies are variants of entropy like EVaR and fuzzy entropy while this study employs Shannon Entropy; however, it can be safely proposed that entropy as a measure of risk is a better risk mitigation measure than the variance

CONCLUSION

The objective of the current study was to investigate the efficiency of the novel Return-entropy portfolio optimization (REPO) method presented by Mercurio et al. (2020) in providing efficient portfolios that are far more diversified than the traditional Markowitz (1952) mean-variance method and giving the optimum portfolios that have better performance and mimic their benchmark more closely than Markowitz's (1952) one.

To fulfil the study's objective, data for three economic blocks, viz., MINT, Visegrád group, and CIVETS, were collected, and portfolios were optimized using both methods. The results indicate that portfolio composition significantly differs using each method, which means portfolio diversification also varies across both methods.

Individual country holdings in the portfolio changed significantly while using variance in the objective function compared to entropy in the objective function. Markowitz's mean-variance portfolio optimization concentrated on a few assets only by assigning them large percentages of weights. On the other hand, REPO does not concentrate on fewer assets and does not allocate large weights to fewer assets in the portfolio.

In addition to assigning different weights to each economic market in the portfolio, both methods differ significantly regarding providing a diversification level. In each case, the return-entropy (REPO) method offered greater diversification in the portfolio. Furthermore, the results are in line with the previous studies of Philippatos and Wilson (1972) Be, and Park (2008), Jana et al. (2009), Samanta and Roy (2005) and, and Kantar (2011) and, Mercurio et al. (2020) who used entropy as an objective function.

The tracking errors of Mercurio & Wu's (2020) return entropy portfolio optimization were lower than those of Markowitz's (1952) mean-variance portfolio optimization. Subsequently, it was also found that the portfolios optimized using the entropy algorithm are more closely mimicking their benchmark and are tighter toward the index. So, from the viewpoint of the investors, the portfolio optimized using return-entropy portfolio optimization provides an efficient tracking of their respective benchmark index compared to the portfolios of Markowitz's (1952) mean-variance method. Therefore, the results concluded that the method presented by Mercurio et al. (2020) is more efficient in real life, providing more diversified portfolios and more closely following their benchmark.

Recommendations and Suggestions

The current study's results yield the following recommendations for investors, portfolio managers, and fund managers: Markowitz's (1952) mean-variance portfolio optimization is one of the most widely used techniques among investors and portfolio managers. However, it assumes returns to be normally distributed. In their study, Xiong et al. (2011) suggested that most portfolios and asset classes are non-normally distributed. Therefore, variance is a poor measure of risk.

Consequently, the current study empirically proved the entropy-based method more efficient than Markowitz's mean-variance method. So, it is advisable to use entropy in the objective function instead of variance because the portfolio optimized using entropy is far more diversified than the traditional method. Also, entropy does not have any distribution assumption. This premise proposes that the investors profitability can be improved with the help of this methodology

Furthermore, the practical difficulties found with the mean-variance method can be overcome using the return-entropy method. It can also preserve the dependence structure. Furthermore, whenever investor views are incorporated, optimal solutions found with REPO are far more stable than the ones found with mean-variance portfolio optimization. Moreover, the risk is directly calculated in the objective function, so there is no need to calculate the covariance matrix or variance of the securities beforehand. Moreover, the method is easier to use as the former method involved the calculation of joint entropy, which complicates the computations in the case of n-assets in the portfolio. Therefore, it is advisable to use return-entropy, which is fully

nonparametric, robust and has no normality assumption. Hence, REPO is the most adequate technique for portfolio optimization and suitable for the investors.

Future Research Direction

The current study can be extended and has some directions for future research. The return-entropy portfolio optimization was investigated using monthly data, but the study can be extended using weekly data. The proposed methodology can be utilized in the formation of the dependent portfolios of the asset pricing models. The hybridization of systematic risk and Liquidity risk with entropy is practicable research endeavour. The REPO methodology can be extended to emulate real estate, cryptocurrencies and commodities. The application of REPO in big data can be investigated with the help of Genetic algorithms for portfolio diversification. The study is limited to only three economic blocks; it can be extended to other economic blocks like BRICS. Furthermore, some other tracking error measures can be used for future research. Extending the work to the micro level is recommended by taking the stock data to check the entropy method's efficiency.

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